The 7th Seminar on Functional Analysis and its Applications
1-2 March 2023
Imam Khomeini International University
Qazvin, Iran

# ON THE ANALYSIS OF A GENERAL NONLINEAR OPERATOR FOR A MODEL IN RANDOM MARKETS

MAHDI KESHTKAR*, ELYAS SHIVANIAN

*Department of Mathematics, Buein Zahra Technical University, Buein Zahra, Qazvin, Iran*
*keshtkarmahdi@gmail.com*
*Department of Applied Mathematics, Imam Khomeini International University, Qazvin 34148-96818, Iran*
*shivanian@sci.ikiu.ac.ir*

Abstract. A generalization of the continuous economic model is proposed for random markets. In this model, agents interact by pairs and exchange their money in a random way, in general, with possibly non-constant total amount of "money". This model takes the form of an iterated nonlinear map of the distribution of wealth. We show the only way to reach equilibrium fixed point distribution is the agents to share their money without expansion or contraction factor.

## 1. Introduction

Recently, based on the idea of pseudo-gases, a control parameter that shows the degree of exchanges between economic factors has been considered in the interval [0,1], that is, if the value of the parameter is zero, there is no interaction between the factors, and if it is one, all factors interact under these conditions. They have reached the conclusion that the Gibbs exponential distribution is established for the mentioned interval [1]. The

2020 *Mathematics Subject Classification.* Primary 47B35; Secondary 30H05
*Key words and phrases.* Equilibrium point, Evolutionary Model, Non-linear Operator, Random Markets.
* Speaker.

relationship and behavior of the moments when they are not convergent have been investigated on the model [2].
Also, to see the convergence of exponential wealth distribution in discrete stochastic markets and their complete analysis, you can refer to reference [3]. A similar model has been proposed for ideal gases, which shows that it converges to the Maxwell distribution, based on which it has been considered as the equilibrium point of the operator with the simulations [4]. The upward growth of entropy for the model in random markets is proof [5]. Following the works, in this article we have proposed a continuous economic model for random markets. In the next section, we state a generalization of the model. Then, we discuss the features of the operator.

## 2. Generalization of Z-model

In this section, we suggest a general form of Z-model that is similar to the behavior of original model (Z-model) in which each of the two partners in a transaction have a random amount $u$ and $v$. During the transaction, they put first the whole amount $(u+v)$ in a basket and then share its content randomly. The new model is defined as

$$P_{n+1}(x) = Tp_n(x) = \iint_{S_{a,b}(x)} dudv \frac{P_n(u)P_n(v)}{au+bv}, \tag{2.1}$$

where, $a$ and $b$ are real positive parameters, and $S_{a,b}(x)$ is defined by the set

$$\{(u,v),\ u,v>0,\ \ x<\ au+bv\}$$

. In this model at the time of the transaction between the two individuals, one of the individual puts $au$ in the basket (instead of $u$ in the Z-model) and the other puts $bv$ in the basket, instead of $v$. As it will be seen, this model is not conservative except when some special conditions hold for the coefficients$a$ and $b$ which will be determined later. If we consider the symmetrical interaction for the pair of agents$(v,u)$, in this case the first agent will put $av$ in the basket and the second one $bu$. For both trades, those of the pairs$(u,v)$and $(v,u)$, the total money to share in the basket is $(a+b)(u+v)$.

## 3. Properties of the operator $T$

First, in order to set up the adequate mathematical framework, we provide the following definitions.

**Definition 3.1.** We introduce the space $L_1^+$ of positive functions (wealth distributions) in the interval $[0,\infty)$,

$$L_1^+[0,\infty) = \{y : [0,\infty) \to R^+ \cup \{0\}, \|y\| < \infty\},$$

with norm-1

$$\|y\| = \int_0^\infty y(x)dx.$$

In particular, consider the subset of $L_1^+[0,\infty)$ i.e. the unit sphere

$$B = \{y \in L_1^+[0,\infty), \quad \|y\| = 1\}$$

**Definition 3.2.** We define the mean richness $< x >_y$ associated to a wealth distribution $y \in L_1^+[0,\infty)$ as the mean value of $x$for the distribution$y$. In the rest of the paper, we will represent it by$< y >$. Then,

$$< y > \equiv < x >_y = \|xy(x)\| = \int_0^\infty xy(x)dx.$$

**Definition 3.3.** For $x \geq 0$and$y \in L_1^+[0,\infty)$ the action of the operator $T$on $y$ is defined by

$$T(y(x)) = \iint_{S_{a,b}(x)} \frac{y(u)y(v)}{au+bv} dudv$$

where$S_{a,b}(x)$ is the region of the plane representing the pairs of agents $(u,v)$ which can generate a richness $x$ after their trading, i.e.

$$S_{a,b}(x) = \{(u,v), u, v > 0, \, au + bv > x\}$$

**Theorem 3.4.** *For any $y \in L_1^+[0,\infty)$ we have $\|Ty\| = \|y\|^2$. In particular, for $y$ being a PDF, i.e. if $\|y\| = 1$, then $\|Ty\| = 1$. (It means that the number of agents in the economic system is conserved in time).*

**Theorem 3.5.** *The operator $T$ is Lipchitz continuous in $B$ with Lipchitz constant $\leq 2$.*

**Theorem 3.6.** *The mean value$< y >$of a PDF $y$ is not conserved in general, that is it would be possible$< Ty > \neq < y >$ for any $y \in B$. (It means that the mean wealth, and by extension the total richness, of the economic system are not preserved in time).*

**Corollary 3.7.** *The mean wealth, and by extension the total richness, of the economic system is preserved in time provided that $a + b = 2$.*

In the next section, it will be revealed that the total richness increases when $a + b > 2$ and decreases when $a + b < 2$.

**Theorem 3.8.** *For any$y \in L_+^1[0,\infty]$, $n \epsilon \mathbb{N}$ and $a, b \in \mathbb{R}^+$it holds*

$$< T^n y > - < y > = ((\frac{a+b}{2})^n - 1) < y > .$$

## References

1. R. Lopez-Ruiz, E. Shivanian, S. Abbasbandy, and J. R. Lopez, *A Generalized Continuous Model for Random Markets*, Mathematica Aeterna,**3** (2013) 317-328.
2. J. L. Lopez, R. Lopez-Ruiz, and X. Calbet, *Exponential wealt distribution in a random market: A rigorous explanation*, J. Math. Anal. Appl., **386** (2012) 195-204.
3. G. Kutriel, *Convergence to the exponential wealth distribution in a discrete-time random market model*, Applicable Analysis, **93** (2014) 1256-1263.
4. E. Shivanian, and R. Lopez-Ruiz, *A new model for ideal gases. Decay to the Maxwellian distribution*, Physica A, **391** (2012) 2600-2607.
5. S. M. Apenko, *Monotonic entropy growth for a nonlinear model of random exchanges*, Phys. Rev. E, **87** (2013) 024101.