STUDY ON NEW SUBCLASSES OF ANALYTIC FUNCTIONS IN GEOMETRIC FUNCTIONS THEORY

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Abstract. In this paper, we give characterization of subclasses of analytic functions, we consider $\Phi-$like functions on the unit disc in $\mathbb{C}$ in terms of Löwner chains.

1. Introduction

The class of all analytic functions was denoted by $\mathcal{A}$. Functions belonging to this class can be displayed in the form of the following power series

$$f(z) = z + \sum_{n=1}^{\infty} a_n z^n. \tag{1.1}$$

which are analytic in the open unit disc $\Delta = \{z : z \in \mathbb{C} : |z| < 1\}$. Further, by $\mathcal{S}$. The class of univalent functions in $\mathcal{A}$ which normalized with the conditions $f(0) = f^{'}(0) - 1 = 0$ was represented by $\mathcal{S}$.
Also, let S^* denote the class of starlike functions that is defined as

$$S^* = \left\{ f \in \mathcal{S};\ Re\ \left(\frac{zf^{'}(z)}{f(z)} \right) > 0,\ z \in \Delta \right\}.$$

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Theorem 1.1. *Let $f : \Delta \to \mathbb{C}$ be a holomorphic function such that $f(0) = 0$ and $f^{'}(0) \neq 0$. Also let $\alpha \in \mathbb{R}$, $|\alpha| < \frac{\pi}{2}$. Then f is spiralike of type α if and only if*

$$Re\left(e^{i\alpha}\frac{zf^{'}(z)}{f(z)}\right) > 0,\ z \in \Delta$$

Definition 1.2. Let f be analytic in the unit disk Δ of the complex plane with $f(0) = 0, f^{'}(0) \neq 0$. Let Φ be analytic on $f(\Delta)$ with $\Phi = 0$, $Re\,\Phi^{'}(0) > 0$. Then f is Phi-like(in Δ) if

$$Re\left(\frac{zf^{'}(z)}{\Phi(f(z))}\right) > 0,\ z \in \Delta \tag{1.2}$$

Remark 1.3. The two clasical case of Definition 1.2 are given by $\Phi(\omega) = \omega(f$ is starlike) and more generally, $\Phi(\omega) = \lambda\omega$, $Re\,\lambda > 0$(f is apiral-like of type argλ).

Definition 1.4. Let $\mathcal{P}$ denote the class of holomorphic functions p in Δ such that $p(0) = 1$ and Re $p(z) > 0, z \in \Delta$.
This class is usually called the Caratheodory class.

In the lemma and theory of Lowner chains, if f is a function which depends holomorphically on $z \in \Delta$ and is also a function of other real variables, it is customary to write $f'(z, .)$ instead of $\frac{\partial f}{\partial z}(z, .)$.

Lemma 1.5. [1] *The function $f : \Delta \times [0, \infty) \to \mathbb{C}$ with $f(0, t) = 0, f'(0, t) = e^t$, is a Lowner chain if and only if the following conditions hold:*
(i) There exist $r \in (0, 1)$ and a constant $M \geq 0$ such that $f(., t)$ is holomorphic on Δ_r for each $t \geq 0$, where $\Delta = \{z \in \mathbb{C} : |z| < r\}$, locally absolutely continuous in $t \geq 0$ locally uniformly with respect to $z \in \Delta_r$, and

$$|f(z, t)| \leq Me^t, \qquad |z| \leq r, \quad t \geq 0.$$

(ii) There exists a function $p(z, t)$ such that $p(., t) \in \mathcal{P}$ for each $t \geq 0, p(z, .)$ is measurable on $[0, \infty)$ for each $z \in \Delta$, and for all $z \in \Delta_r$,

$$\frac{\partial f}{\partial t}(z, t) = zf'(z, t)p(z, t), \qquad a.e. \quad t \geq 0.$$

(iii) For each $t \geq 0, f(., t)$ is the analytic continuation of $f(., t)|_{\Delta_r}$ to Δ, and furthermore this analytic continuation exists under the assumptions (i) and (ii).

Lemma 1.6. *Let $f(z, t)$ be a Lowner chain. Then there exists a function $p(z, t)$ such that $p(., t) \in \mathcal{P}$, $t \geq 0$, $p(z, t)$ is measurable in $t \in [0, \infty)$ for each $z \in \Delta$, and*

$$\frac{\partial f}{\partial t}(z, t) = zf'(z, t)p(z, t), \quad z \in \Delta, t \geq 0. \tag{1.3}$$

some researchers investigate on subclasses of univalent functions by using of new methods [2, 5, 3, 4].
In this paper, we introduce new subclass of univalent functions, also we shall obtain characterization of the subclass by using the lowner chains method.

2. Main Results

Definition 2.1. Let f be analytic in the unit disk Δ of the complex plane with $f(0) = 0, f^{'}(0) \neq 0$. Let Φ be analytic on $f(\Delta)$ with $\Phi = 0$, $Re\,\Phi^{'}(0) > 0$. Then f is almost Φ-like function of order α(in Δ, $0 \leq \alpha < 1$)if

$$Re\ \left(\frac{\Phi(f(z))}{zf^{'}(z))}\right) > 0,\ z \in \Delta \tag{2.1}$$

Theorem 2.2. *Let f is Φ-like in Δ. Then f is univalent in Δ and $f(\Delta)$ is Φ-like.*

Corollary 2.3. *Let f be analytic in Δ with $f(0) = 0$. Then f is univalent in Δ if and only if f is Φ-like for some Φ.*

Theorem 2.4. *Let D be simply connected subset of $\mathbb{C}$, so suppose $f \in \mathcal{A}$ be a Φ-like function on Δ with $\Phi^{'}(0) = 1$ and let $f(z) \in D$ if and only if $f(z,t) = e^t\Phi(f(z))$ is a lowner chain.*

Theorem 2.5. *Let $f \in \mathcal{A}$ be a Φ-like function on Δ with $\Phi^{'}(0) = 1$, $\omega \in f(\Delta) - 0.$, if and only if $f(z,t) = e^t f(z)$ is a lowner chain.*

Theorem 2.6. *Let D be simply connected subset of $\mathbb{C}$, so let $f \in \mathcal{A}$ be a almost Φ-like function of order α on Δ with $\Phi^{'}(0) = 1$ and let $f(z) \in D$ if and only if*

$$g(z,t) = e^{\frac{1}{1-\alpha}t}\Phi(f(e^{\frac{\alpha}{\alpha-1}t}z)), \quad (0 \leq \alpha < 1,\ z \in \Delta, t \geq 0)$$

is a lowner chain.

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