**Theory of in-plane vibrations of nano-mechanical disk resonators in liquids**

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**Abstract**

Characterization of liquids through their shear rheological properties is a well-established approach in different fields of research and industrial applications. Micro-electromechanical systems (MEMS) are among the best solutions for rheology at frequencies up to 100 MHz. Nanomechanical resonators enable exploring the rheological properties of liquids at higher frequencies. Here, we report on an analytical investigation on the solid-liquid interaction for the in-plane vibration of an elastic disk surrounded by a Newtonian liquid. For water, we show that at high frequencies, acoustic radiation is the dominant mechanism in energy loss while at lower frequencies the shear friction dominates. We numerically examine these findings by using the finite element method (FEM) in liquid water at frequencies between 200 MHz~ 2GHz.

**Keywords:** rheometry, shear dynamic, nano-optomechanics, Newtonian liquid.

**تحلیل ارتعاشات درون صفحه‌ای تشدیدکننده دیسک نانومکانیکی در حضور سیالات**

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# چكيده

شناسایی مایعات از طریق خواص رئولوژیکی برشی رویکردی تثبیت شده در زمینه های مختلف تحقیقاتی و کاربردهای صنعتی است. سیستم های میکرو الکترومکانیکی (MEMS) یکی از بهترین راه حل ها برای رئومتری در شرایط تعادل در فرکانس های کمتر از100 مگاهرتز هستند. تشدیدکننده‌های نانومکانیکی، مطالعه خواص رئولوژیکی مایعات در فرکانس‌های بالاتر را امکان‌پذیر ساخته‌اند. در این مقاله، روند مدلسازی تحلیلی برهمکنش جامد- سیال در حالت ارتعاش درون صفحه‌ای تشدیدکننده الاستیکی دیسک در حضور مایع نیوتنی بیان شده است. نشان داده شده‌است که در حالت ارتعاش درون آب، با افزایش فرکانس تشدید، انتشار امواج صوتی عامل غالب در تلفات انرژی خواهد بود در حالی که در فرکانس‌های پایین‌ ویسکوزیتی برشی عامل اصلی است. صحت نتایج به دست آمده، با مدلسازی عددی روش المان محدود در محدوده فرکانسی 200 مگاهرتز-2گیگاهرتز مورد ارزیابی قرار گرفته است. تطابق خوب نتایج حاصل از این دو روش، بیان‌کننده صحت مدل تحلیلی ارائه شده در این مقاله است.

**کليدواژه­ها:** رئومتر، تنش برشی، اپتومکانیک، سیال نیوتونی.

**Introduction**

Direct mechanical probing of liquids is among the best solutions to investigate both the rheological properties of liquids in equilibrium conditions and to explore solid-liquid interaction dynamics[1]–[4]. However, there is no direct method operating at frequencies between 100 MHz and 7GHz. Recently, a suspended microdisk optomechanical resonator vibrating in the plane has been introduced to probe the shear dynamics of liquids in this range of frequency[5]. However, the lack of global fluid-structure model in such configuration hinders the inference of the liquid properties from experimental measurements. Here, we present a theoretical framework to study the solid-liquid interaction of an in-plane vibrating microdisk immersed in a Newtonian liquid environment (Fiq.1a). It is obtained through a perturbative mode-matching approach: first, the resonator vibration profile is calculated in vacuum considering free boundary condition; second, the velocity of the fluid is calculated by solving Navier-Stokes equations and imposing continuity of velocity at the solid-liquid interfaces. Finally, the vibrational spectrum of the disk in presence of the liquid is calculated.

**Theoretical Modeling**

Total vibration velocity profile of the fundamental in-plane mode of a disk resonator in vacuum environment can be expressed as:

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|  | (1) |

Where , ρs is density, E is Young modulus, and ν is Poisson ratio. The radial component of velocity is shown in Fig.1a. By imposing free boundary conditions, the corresponding characteristic equation is obtained:

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|  | (2) |

and allows us obtaining resonant vibration frequencies. Following our perturbative approach, we next calculate the velocity field at the interface of solid and liquid by using Navier-Stokes equations:

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| --- | --- |
|  | (3) |
|  | (4) |

where p is pressure, µ is shear viscosity, µB is bulk viscosity, and β=(ρc2)-1 is compressibility, ρ is the density and c is the speed of sound in liquid.

In order to have an analytical expression for the velocity, we express the fluid velocity vector as the sum of a divergence-free and a curl-free terms **v**(**r**)= **∇×Ψ**+**∇**ϕ. Replacing these two orthogonal terms into the Eq.3 leads to two independent equations:

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|  | (5) |
|  | (6) |

Using continuity of velocity at the interfaces in cylindrical coordinates, we determine the velocity and stress of the fluid in the presence of the disk. With these elements at hand, we can compute the vibration spectrum of the disk resonator dressed by its interaction with the liquid.

**Results and Discussion**

Here, we study the case of silicon disks with their radius varying between 2um~10um and thickness of 220 nm. We consider water with shear viscosity 1mPas, density 1000kg/m3, and speed of sound 1500m/s. Fig.1b, and 1c, show good agreement between our theoretical modeling and the numerical study. Using our analytical model, it can be shown that the dissipation for larger disks (smaller frequencies) is mainly determined by shear friction, while for the smaller disks (higher frequencies) the compressibility and acoustic radiation become the dominant mechanism.

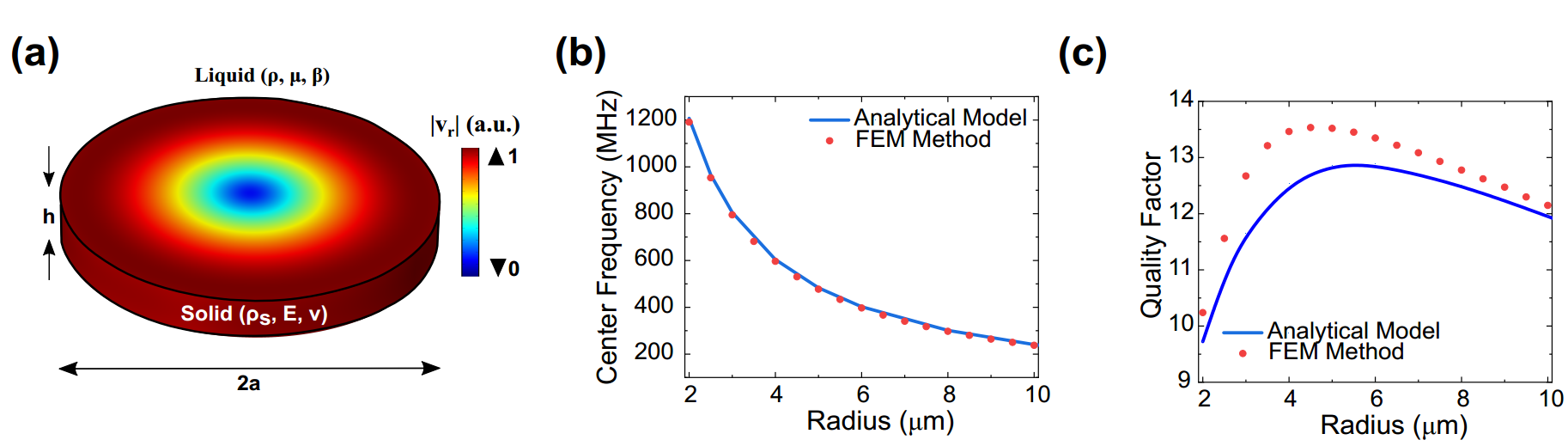


Figure 1: Fluid-disk interaction in a Newtonian liquid. (a) Distribution of radial velocity vr for the fundamental in-plane mode of the disk resonator. (b), (c) Analytical (solid line) and numerical FEM (circles) calculations of the center frequency and quality factor of the fundamental in-plane vibration mode of the disk. Thickness of the disks = 220nm.

**Conclusion**

In this paper, we have presented a theoretical framework to study the frequency response of the fundamental in-plane vibration mode of miniature disk resonators immersed in the Newtonian liquid. The validity of the model has been verified by comparing it with numerical FEM calculations.

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